# A Benchmark Test for Parallel Algorithmic Differentiation

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# Outline

Introduction to AD

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5. $z^0 = \log(s^0)$
6. Output: dependent variable values $(u^0, v^0, s^0, z^0)$

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$$z^1 = \frac{\partial z}{\partial x}(x^0, y^0)x^1 + \frac{\partial z}{\partial y}(x^0, y^0)y^1$$

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5. $z^1 = s^1/s^0$
6. Output: $(u^1, v^1, s^1, z^1)$

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$$(x^1, y^1) = \left[ z^1 \frac{\partial z}{\partial x}(x^0, y^0), z^1 \frac{\partial z}{\partial y}(x^0, y^0) \right]$$

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4. $y^1 = v^1 \exp(y^0)$
5. $x^1 = u^1 \exp(x^0)$

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6. Output: $(x^1, y^1)$

# AD and Global Variables

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# Example Use of Multi-Newton Benchmark

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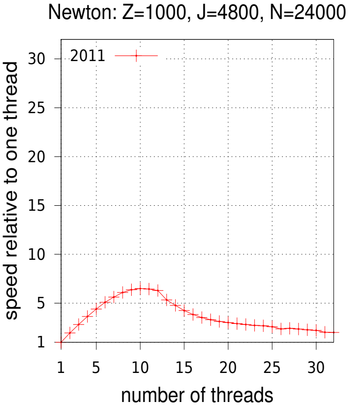

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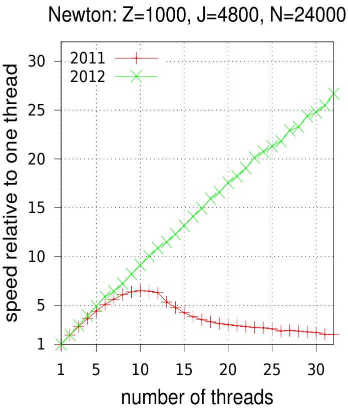

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5. If $f(x_k)f'(x_k) \geq 0$ and $x_k = a$, output $B = \emptyset$.

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9. Set $k = k + 1$. Goto step 3

# Multi-Newton Setup

Set $S(\alpha, \beta, K, \delta, J, M, f)$ to multiple zeros in $[\alpha, \beta]$ :

1. Input: bounds $[\alpha, \beta]$, maximum iterations $K$, criterion $\delta$, number of sub-intervals $J$, number of threads $M$, function $f$.

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2. Set sub-interval length $\gamma = (\beta - \alpha)/J$, start for first thread $s_1 = \alpha$, end for last thread $e_M = \beta$.

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3. For $m = 1, \ldots M$, in sequential mode execute:

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3. For $m = 1, \ldots M$, in sequential mode execute:
   3.1 Number of sub-intervals for thread $m$
   $$L_m = \begin{cases} \text{floor}(J/M) + 1 & \text{if } m \leq \mod(J, M) \\ \text{floor}(J/M) & \text{otherwise} \end{cases}$$

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   - 3.2 Start for thread $m \geq 2$, $s_m = s_{m-1} + \gamma L_m$.
   - 3.3 End for thread $m - 1 \geq 1$, $e_{m-1} = s_m$.

# Multi-Newton Continued

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4. For $m = 1, \ldots, M$, in parallel mode execute:
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5. Set $S = \cup_{m=1}^{M} S_m$.
6. If difference between two elements of $S$ is less than $\gamma/2$, remove one that has larger value for $|f(x)|$.
7. Output $S$.

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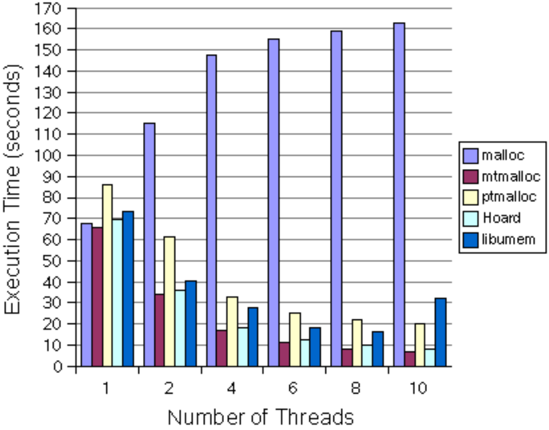
  Each call runs *num_threads* versions of *worker* with corresponding $\mathrm{thread\_num}()$ between 0 and $M - 1$.
- $ok = \mathrm{team\_destroy}()$
  terminates all but the master thread; i.e., $m = 0$.

# Motivation

The following plot is taken from
A comparison of memory allocators in multiprocessors:



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**2012.0** For each thread $m$ and each capacity $i$, a singly linked list of available memory is retained with root $A_{m*I+i}$. In addition, counter vectors for the number of bytes inuse $u_m$ and available $a_m$ for corresponding thread.

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**2012.1** For each thread $m$, a separate structure $S_m$ is allocated for the inuse counter, available counter, and vector of available roots for that thread.

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$\beta$ Upper search limit $\beta = (Z-1)\pi$.

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# Benchmark Parameters Choices

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$$f(x) = \frac{1}{N}\sum_{n=1}^{N}\sin(x)$$

$N$ Work load per function evaluation.

$Z$ Number of zeros $Z = 1000$.

$\alpha$ Lower search limit $\alpha = 0$.

$\beta$ Upper search limit $\beta = (Z - 1)\pi$.

$\delta$ Convergence criterion: $\delta = \beta \cdot 10^2 \cdot \varepsilon$.

IHME

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$J$ Number of bounded Newton sub-intervals $J = 4800$.

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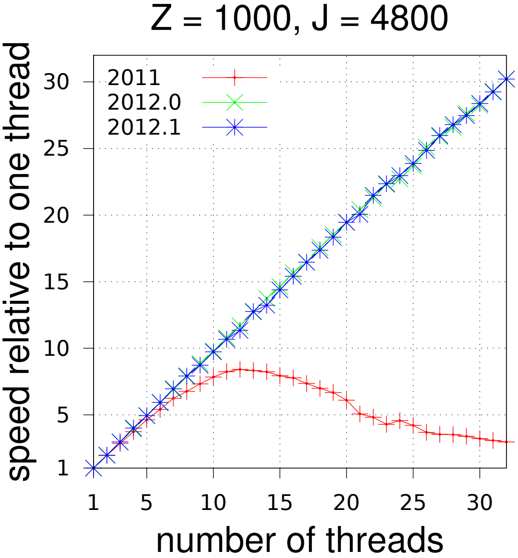
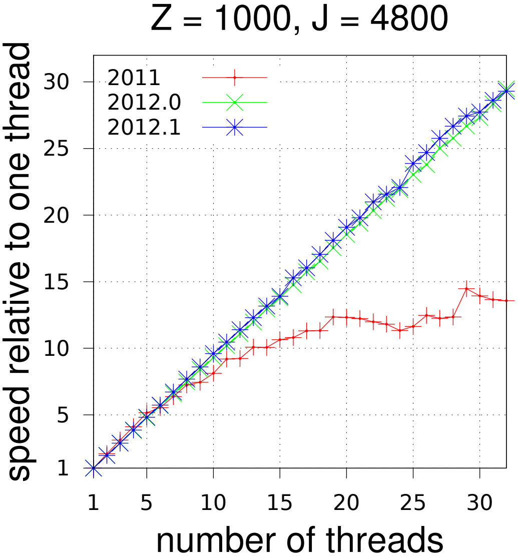
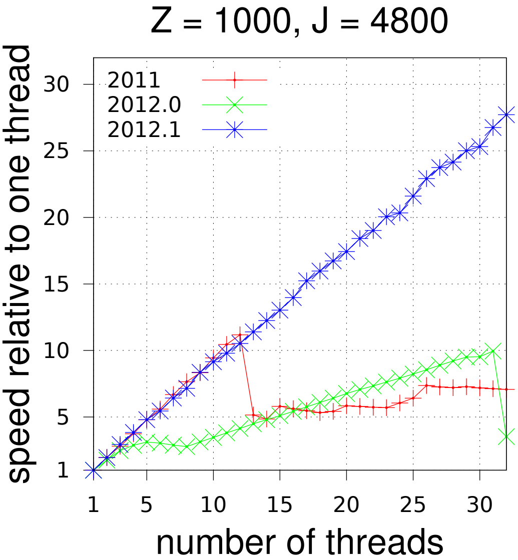
# 32 cores, $N = 10000$



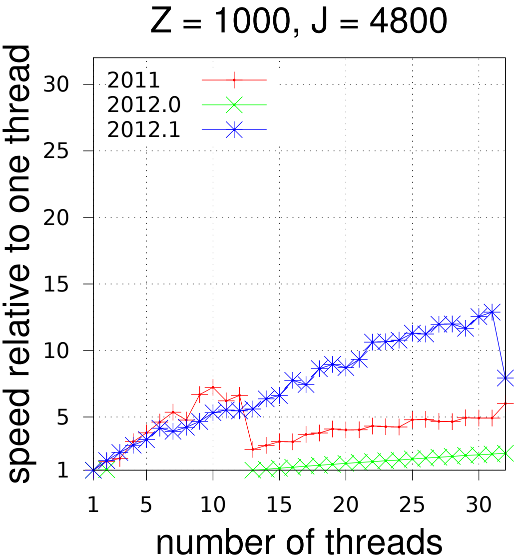
# 32 cores, N = 1000



## 32 cores, N = 100



# 32 cores, N = 10



Z = 1000, J = 4800
2011
2012.0
2012.1
speed relative to one thread
number of threads
1
5
10
15
20
25
30